SIMILARITY PARAMETERS FOR THE EXPANSION OF A SUPERSONIC JET IN A CHANNEL WITH A SUDDENLY CHANGING CROSS SECTION

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By a generalization of numerous experimental data, the similarity parameters are found with which the jet discharge from a nozzle into a channel with a suddenly changing cross section can be simulated.

A great many engineering devices involve a supersonic jet flowing within a confined space. A theoretical analysis of this phenomenon is difficult, since the processes occurring here are very complex and also because the physical mechanism here is not clearly enough understood. A huge quantity of diverse experimental data cannot be generalized properly, as long as the necessary similarity criteria have not been established. For several reasons, the methods of similarity theory and dimensional analysis (e.g., the π-theorem) do not yield satisfactory results when applied to the specific instances of interest here. Any similarity criteria obtained for the expansion of a jet into a confined space will, therefore, be very important.

The flow of a single or multiple (bundled) supersonic jet along channels with a cylindrical or a suddenly changing cross section, as shown schematically in Fig. 1, has been analyzed in [1, 2, 3, 4]. The purpose of such devices is to maintain a sufficient degree of rarefaction in the exit region of the nozzle. The jet in case A or the jet bundle in case B enters into a hermetically sealed chamber at the other end of which there is a cylindrical diffuser. In cases C and D the jet is discharged into a cylindrical channel. Further, with $\bar{l}$ = 0, the jet flows through a chamber with a thin diaphragm at the other end.

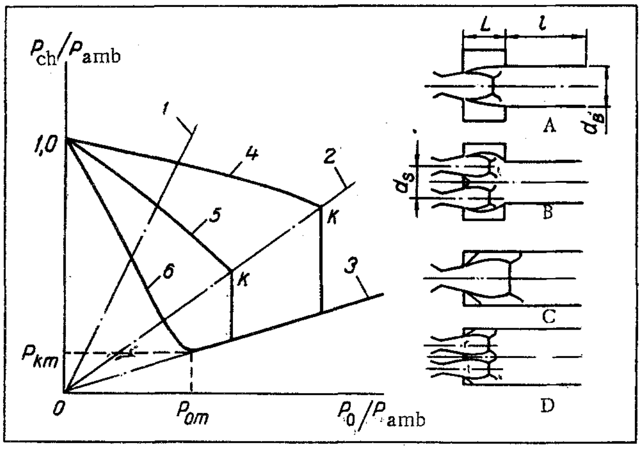

Fig. 1. Typical curves of pressure P_{ch} in the space around a jet entering into a channel with sudden cross section changes, as a function of pressure P_0 at the nozzle entrance, for different geometries of the analyzed channel model: $n = n_{sep}$ (1); $n = n_{ejec}$ (2); $n = n_{indet}$ (3); $\bar{l} = 0$ (4); $\bar{l}_0 \geq \bar{l} \geq 0$ (5); and $\bar{l} \geq \bar{l}_0$ (6).

The problem is to determine the pressure P_{ch} (in the chamber space outside a jet at the exit section of the nozzle) which determines the nozzle discharge characteristics. In addition, one must also know the device geometry which will yield the same pressure in the chamber.

The dependence of pressure P_{ch} on pressure P_0 at the nozzle entrance has been analyzed in [1, 2] for various configurations. A typical curve representing this relation is shown in Fig. 1. As is evident here, the shape of this curve depends on the diffuser or tube length. For a certain diffuser length $\bar{l} = \bar{l}_0$, which depends on the Mach number M_a [2], the falling range of the curve corresponds to ejection and is smooth. When $\bar{l} \geq \bar{l}_0$, the chamber pressure P_{ch} attains its lowest value possible for the given geometry. Beyond the minimum point P_{ch} becomes proportional to pressure P_0 (critical indeterminate

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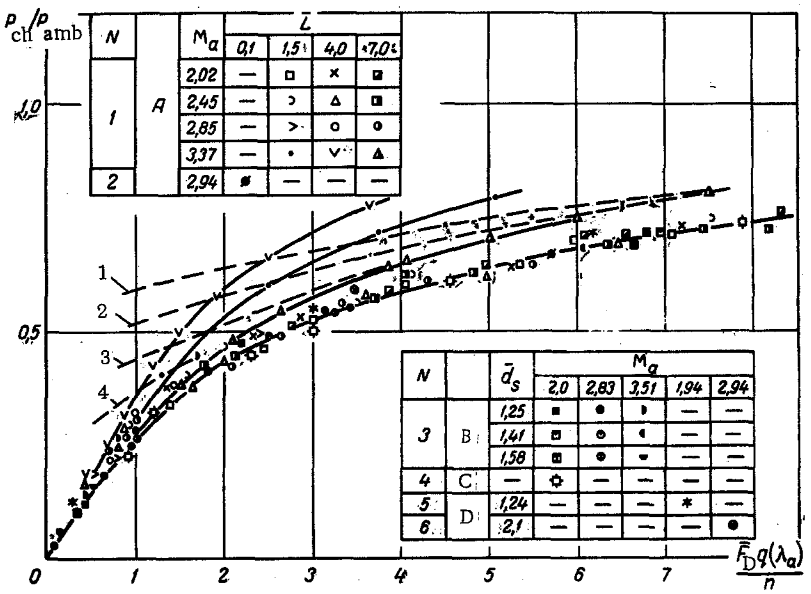

Fig. 2. Pressure P_{ch} in the chamber space around a jet discharging by ejection into a channel with a suddenly changing cross section, as a function of the similarity parameter $\varkappa_l$ for $\bar{l} \geq \bar{l}_0$: n = 0.45 (1); 0.2 (2); 0.3 (3); 0.4 (4).

discharge mode, n_{cr}). It is well known that the value of n_{cr} does not depend on the diffuser length. It can be determined theoretically by the method given in [3]. Pressure P_{0m} can also be calculated, by the method given in [4]. The falling portion of the curve has not been calculated yet and, if necessary, experiments must be performed for each specific system geometry.

When $\bar{l}_0 \geq \bar{l} \geq 0$, the relation $P_{ch} = f(P_0)$ becomes more complicated. The curves are precise within two ranges. In range 1 P_{ch} decreases continuously as P_0 increases till point K is reached, where a condition of instability prevails. Further, P_{ch} decreases at P_0 = const till $n = n_{indet}$. This range is transitional and has been thoroughly described in [1].

Thus, there is no known method for calculating the portions of these curves which correspond to the range $0 < n < n_{ejec}$.

The range of nozzle discharges into the chamber after ejection ceases and before the mode becomes indeterminate, corresponding to a separation of the stream from the nozzle walls, is indicated in Fig. 1. The performance of the given devices when the stream separates from the nozzle is of no practical interest. As has been mentioned already, the portion of the curve for $\bar{l} = \bar{l}_0$ and $n_{sep} < n < n_{ejec}$ has a shape which is different for different values of $\bar{L}$, M_a, and d_{ex}. It appears, however, that it is possible to define a parameter

$$\varkappa_l = \frac{\bar{F}_D q(\lambda_a)}{n},$$

with which these portions of the $P_{ch} = f(P_0)$ curves will be identical for any geometry and any Mach number M_a. If $\varkappa_l$ = const, in other words, then also the chamber pressure P_{ch} = const. This is expressed by the curve $P_{ch} = f(\varkappa_l)$ in Fig. 2, where the results of studies on several model variants are shown. As can be seen here, pressure P_{ch} variation as a function of $\varkappa_l$ is the same for single-nozzle and multinozzle devices. We will note that F_a in the expression for $\varkappa_l$ is, in the case of multinozzle devices, the combined exit section area of all the nozzles.

As Fig. 2 indicates, at M_a = 3.37 the pressure P_{ch} depends on the geometrical parameter $\bar{L}$. This is explainable by the fact that in this case the stream separates from the nozzle ($n < n_{sep} \approx$ 0.5–0.6). The dashed lines on the diagram cover points which correspond to n = const. The curves merge when the indeterminacy of discharge exceeds n_{sep}. Experimental data confirm that, during ejection with $\bar{l} = \bar{l}_0$, the pressure P_{ch} depends only on the parameter $\varkappa_l$ when single or bundled supersonic jets flow along a cylindrical channel or along a cylindrical channel terminating into a chamber. In the test series described here we checked the scale effect. Thus, the ratio of nozzle exit areas in variants 1 and 2 (see the table in Fig. 2) was equal to 39. In both cases the test points fell on the same curve.

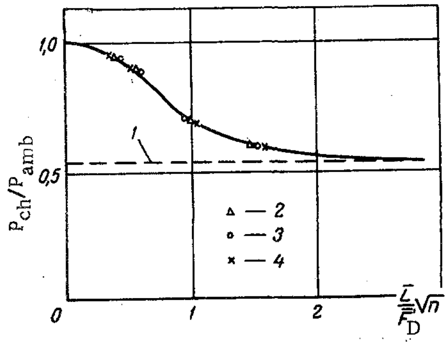

Fig. 3. Pressure P_{ch} in the chamber space around a jet discharging by ejection into a channel with a suddenly changing cross section, as a function of the similarity parameter $\varkappa_0$ for $\bar{l} = 0$: $(P_{ch}/P_{amb})_{cr} = 0.528$ (1); n = 0.5 (2); 1.0 (3); and 2.0 (4).

We have considered one extreme case, where the diffuser length $\bar{l} = \bar{l}_0$ was such as to ensure a maximum rarefaction in the chamber. We will now consider the other extreme case, where there is no diffuser, i.e., $\bar{l} = 0$.

As follows from Fig. 3, the similarity parameter here is

$$\varkappa_0 = \frac{\bar{L}}{\bar{F}_D}\sqrt{n}.$$

The chamber pressure P_{ch} does not depend on the Mach number of the jet at the nozzle exit and the similarity parameter is a purely geometrical quantity. This fact can be easily explained. Indeed, it is well known that for $\bar{l} < \bar{l}_0$ there appears an annular backward stream flowing from the surrounding atmosphere into the chamber through an annular clearance between the forward stream boundary and the flanging diaphragm. This backward stream compensates the adjacent mass of the jet along the distance $\bar{L}$. Within this annular clearance the backward stream accelerates to a certain velocity while its pressure drops from atmospheric to that in the chamber (P_{ch}). We also know that $r_{jet} \sim \sqrt{n}$. Therefore, an increase of $\bar{L}$ or n as well as a decrease of F_D results in a reduction of the area across which the backward stream flows. This in turn increases the velocity of the backward stream and thus decreases P_{ch}, as can be seen in Fig. 3. Since velocity of the backward stream cannot be higher than the velocity of sound, hence $P_{ch} \geq 0.528\ P_{amb}$ and this can also be deduced from the graph.

Thus, considering the stipulation that $\varkappa_l$ be maintained constant for $\bar{l} = \bar{l}_0$ or $\varkappa_0$ be maintained constant for $\bar{l} = 0$, pressure P_{ch} also remains constant. In order to determine P_{ch} during discharge by ejection for $\bar{l} = \bar{l}_0$ and for $\bar{l} = 0$, moreover, one now may use the two graphs (Fig. 2 and Fig. 3) rather than a huge number of test curves. With the aid of relation $P_{ch} = f(\varkappa_l)$ one can also determine the minimum possible pressure P_{ch}, after having first found the value of n_{indet} from the data in [3].

NOTATION

P_0	is the pressure at the nozzle entrance;
P_s	is the static pressure at the nozzle exit;
P_{amb}	is the ambient pressure;
P_{ch}	is the chamber pressure;
$n = P_s/P_{ch}$	is the discharge indeterminary index;
n_{sep}	is the indeterminary index when the jet separates from the nozzle walls;
n_{ejec}	is the indeterminary when ejection ceases;
n_{indet}	is the critical indeterminary index;
u	is the discharge velocity;
a_*	is the critical velocity;
M_a	is the Mach number at the nozzle exit;
d_a	is the diameter of the nozzle exit section;
d_D	is the diffuser diameter;
F_a	is the area of the nozzle exit section;
F_D	is the area of the diffuser entrance section;
L	is the length of the chamber;
l	is the length of the diffuser;
d_s	is the equivalent diameter of bundle of nozzles;
r_{jet}	is the radius of jet boundary;

$\lambda_a = u_a/a_*$;

$$q(\lambda_a) = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}}\left(1 - \frac{k-1}{k+1}\lambda_a^2\right)^{\frac{1}{k-1}};$$

$\bar{F}_D = F_D/F_a$;

$\bar{L} = L/d_a$;
$\bar{l} = l/d_D$;
$\bar{d}_s = d_s/d_a$.

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